Numerical Mathematics 2, test 1
September 30, 2021

Duration: 1 hour and 15 minutes

In front of the questions one finds the points. The sum of the points plus 1 gives the end mark of the test.

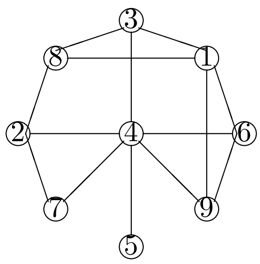
1. Consider the linear problem and the inverse of the occurring matrix:

$$\begin{bmatrix} 10^{-10} & 10^{-5} & 1 \\ 10^{-5} & 1+10^{-10} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1-10^{-10} & -10^{-5}+10^{-15} \\ 1 & -10^{-5}+10^{-15} & -10^{-20} \end{bmatrix}.$$

   We want to solve this on a computer which uses a unit round-off error of 1e-16.

   (a) [0.2]Determine the condition number of the matrix in the infinity norm.
   (b) [1.5] Give the solution if we use Gaussian elimination without pivoting.
   (c) [1.0] Give the solution if we use Gaussian elimination with partial pivoting.
   (d) [0.3] Which of the two approaches gives the correct approximate solution and why?

2. Consider the graph of a symmetric matrix A depicted below.



   (a) [0.2] Make a sketch of the associated matrix (with * if element in matrix is non-zero)
   (b) [1.4] Determine the Cuthill-mcKee ordering and sketch the matrix after reordering.
   (c) [1.4] Give the Reverse Cuthill-mcKee ordering and sketch the occurring L resulting from a Cholesky factorization of that matrix.

3. Consider the following linear system

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

   (a) [0.5] Show that the matrix admits a singular value decomposition (SVD) of the shape $A = UFW^T$ where U and F are matrices of size 2×2 and W of size 3×2.
   (b) [1.5] Compute the SVD factorization of the previous part. (Note that AA^T is diagonal).
   (c) [0.4] Compute the pseudo inverse from A using the SVD.
   (d) [0.6] Observe that $[0,\ 1,\ 0]^T$ is a solution of the above system and that $[1\ ,-1,\ -2]$ is in the kernel of A. Show without using the actual pseudo-inverse, that the solution when using the pseudo inverse, is $[1,\ 5,\ -2]^T/6$.